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MOOC@TU9 Lecture Course Network
Lecture "Orbital Mechanics – Basics"
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Solution "Task of the Week"

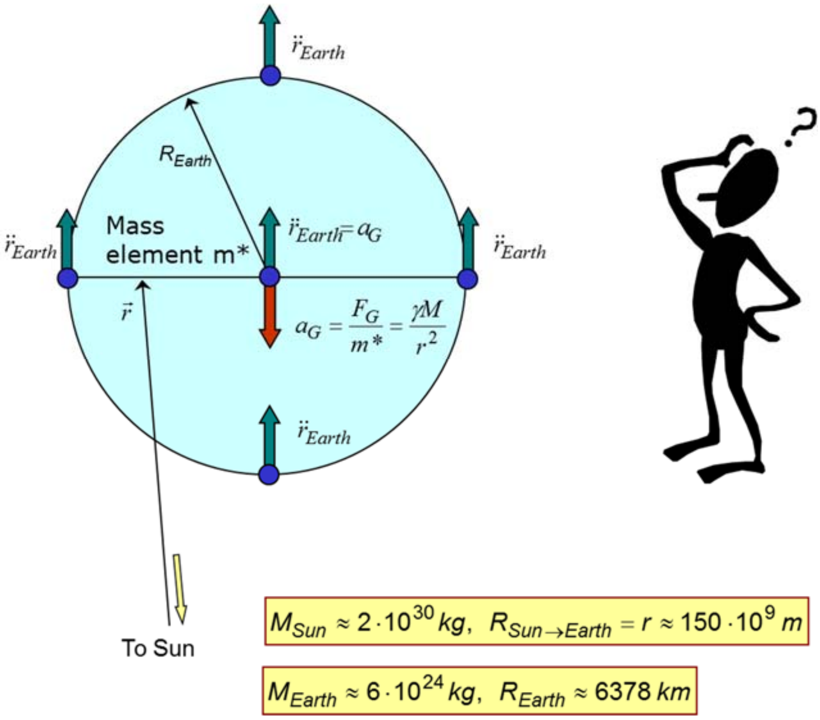


Fig. 1: Task of the week:
Do we "feel" the gravitational pull of the Sun on Earth?

It is tempting to answer that the attracting force (gravity) is counterbalanced by the centrifugal force, i.e. no net force is acting on the Earth (see Fig. 2). Thus, we do not feel any effect. However, this answer is not really correct. As discussed in the lecture, according to Newton's first law, a body would have a constant linear motion if no net force is acting! Now, we obviously cannot claim that the Earth is orbiting the Sun on a straight line!

Indeed, the centrifugal force is just a so-called "pseudo" or "virtual" force, which an accelerated observer "experiences", however this force has no physical origin!

The correct explanation in Newtonian mechanics is given by the 2-body problem (Fig. 3). The only equations we need are given by Newton's second law of motion and by the universal gravitational law (see lecture).

Thereafter, our Earth with mass *m* experiences an acceleration in the direction of the attracting force given by the mass of the Sun M, i.e.

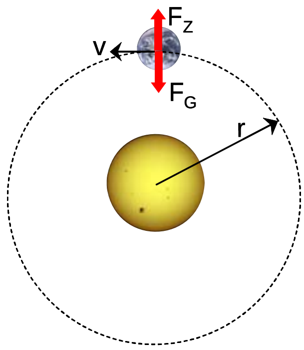


Fig. 2: Centrifugal and gravitational force.

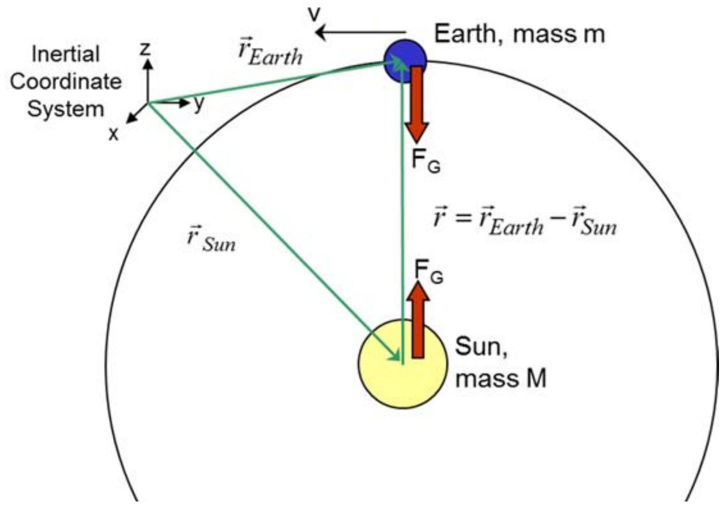


Fig. 3: Explanation of the 2-body equation of motion.

$$\vec{F}_G = -\gamma \frac{mM}{r^3}\vec{r} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt} = m\ddot{\vec{r}}_{Earth} \quad ,$$

where $\vec{r}$ denotes the relative distance between Sun and Earth, and $\ddot{\vec{r}}_{Earth}$ the Earth's acceleration in an inertial coordinate system. Our Earth and its entire inhabitants therefore accelerate continuously towards the Sun, i.e. Earth is in a continuous "free fall" towards the Sun. This acceleration of about

$$\ddot{\vec{r}}_{Earth} = -\frac{\gamma M}{r^3}\vec{r} \approx 0{,}0059\,\frac{m}{s^2}$$

compensates the attracting force. Of course, also the Sun is accelerated (action = reaction), i.e.:

$$-\vec{F}_G = M\ddot{\vec{r}}_{Sun} \quad ,$$

or

$$\ddot{\vec{r}}_{Sun} = \frac{\gamma m}{r^3}\vec{r} \approx 1{,}8 \cdot 10^{-8}\,\frac{m}{s^2} \quad ,$$

which is obviously negligible in comparison to the Earth's acceleration. Thus, the overall equation of motion reads:

$$\ddot{\vec{r}} = \ddot{\vec{r}}_{Earth} - \ddot{\vec{r}}_{Sun} = -\gamma\frac{M}{r^3}\vec{r} - \gamma\frac{m}{r^3}\vec{r} = -\gamma\frac{(M+m)}{r^3}\vec{r} \quad .$$

In summary:

- Gravitational pull of the Sun is compensated by a continuous “free-fall“ towards Sun.
- Acceleration occurs (almost) perpendicularly to Earth’s own velocity, changing continuously the direction of the velocity vector.
- Acceleration is such that the amount of Earth’s velocity remains (almost) unchanged.

  → The entire Earth is in a continuous free-fall around the Sun!

  (more precisely: around the common centre of masses).

In other words: The Earth falls always towards the Sun, however because of the Earth’s own velocity, the curvature of the falling path is such that the Earth (fortunately) never touches the Sun’s surface.

An implicit assumption in this discussion is that both the Earth and the Sun represent point masses. Thus, the question arises what happens if we consider the spatial extension of the Earth.

First, we may assume that every point on (and in) Earth is rigidly connected to the Earth’s centre. Thus, every mass point experiences the same “falling” acceleration in an inertial co-ordinate system. However, the distance of every point mass to the Sun is different, in direction and/or amount (Fig. 4). This difference leads to the so-called gravity gradient which acts on a mass element. On Earth’s surface, the gravity gradient reaches a maximum value of about

$$a_{GG} \approx 2\frac{\gamma M}{r^3}R_0$$

in the direction of the connecting line between Earth and Sun and a minimum value of about

$$a_{GG} \approx \frac{\gamma M}{r^3}R_0$$

perpendicular thereto (as depicted in Fig. 4)

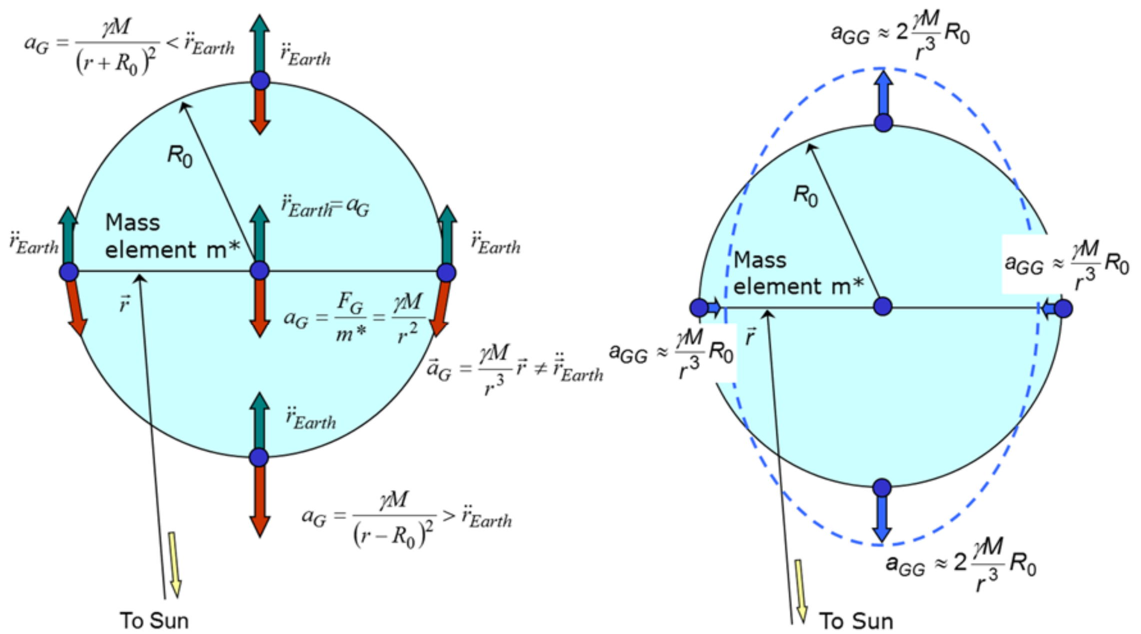


Fig. 4: Explanation of the gravity gradient.

It is now interesting to discuss the values of the acceleration caused by the gravity gradient in comparison to other values:

- Earth gravitational acceleration for a spherical Earth ($R_0 = 6378\,km$): $g_0 = 9.81\,m/s^2$
- For comparison, in low Earth orbit in 300 km altitude($R_0 = 6678\,km$): $g = 8.94\ m/s^2$
- Due to Earth's own rotation: 0.0339 m/s², value at the equator, decreasing with increasing latitude, vanishes at the poles
- Sun's gravity gradient on Earth's surface (max.): 0.000 000 5 m/s², i.e. relative to g_0 about 0.05 ppm.
- Because Earth's own gravitation also changes with the distance to its centre, this value corresponds to an altitude change on Earth's surface of about 0.15 m.

As a side note:

- Gravity gradient of the Moon (max.): 0.000 001 1 m/s², i.e. relative to g_0 ca. 0.11 ppm.

In summary: Earth is in a continuous free-fall around the Sun. Thus, the mass of the Sun has an effect on the period of the Earth, i.e. the duration of "one year". One could maybe "feel" the occurring gravity gradient as an overlying periodic change of Earth's own gravity force (if we would have a receptor capable to detect changes in the "parts per billion"" ppb-regime). Nevertheless, effects of the gravity gradient are certainly of importance (e.g. tides, see Fig. 5) and might be perceptible for example during a walk along the beach of a steep coast…

Many other effects in our solar system may also be explained by the occurring gravity gradients, e.g. volcanism or the synchronous rotation of some moons (the latter also for Earth's moon) as sketched in Figs. 6 and 7.

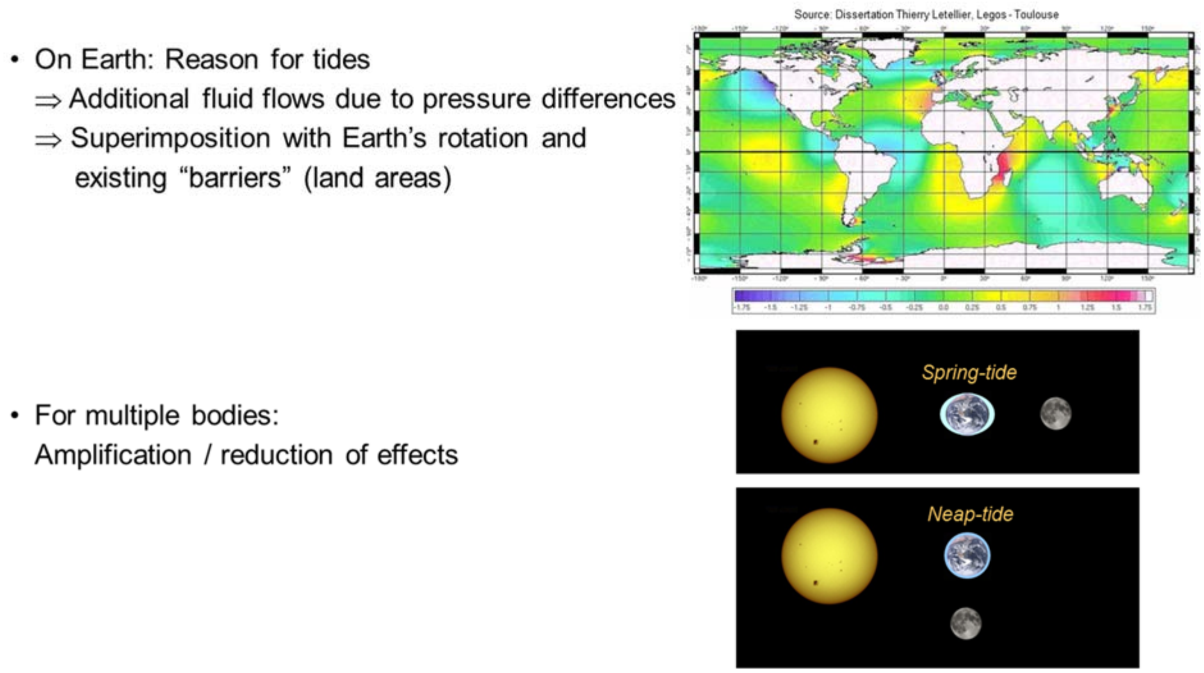


Fig. 5: Effect of the gravity gradient: Tides.

- Earth rotates faster than Moon revolves
  ⇒ Moon obtains acceleration component in orbit direction
  ⇒ Increase of Moon's orbit at the expense of Earth's rotational energy
- Gravity gradient affects also Moon
  ⇒ Because Moon is the smaller body, faster reduction of own rotation until synchronous rotation

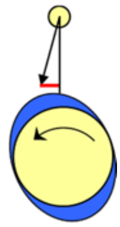
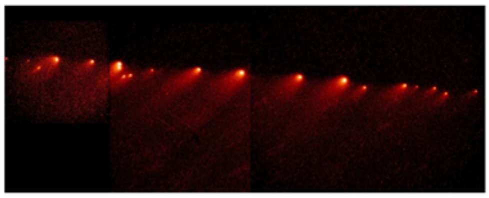
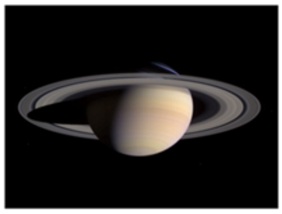


Fig. 6: Effect of the gravity gradient: Synchronous rotation of smaller bodies.

- Comet Shoemaker-Levy
  - Passed Jupiter in July 1992 and burst in 21 "pieces"
  - Between 16-22 July 1994: Fragments struck Jupiter's surface
- Saturn's rings were believed to be within "Roche-limit", where effect of gravity gradient is higher than a potential gravitational force of a (smaller) body (note: other explanation nowadays...)
- Volcanic activity of Io caused by Jupiter's gravity gradient (because of elliptical orbit, although synchronous rotation!)

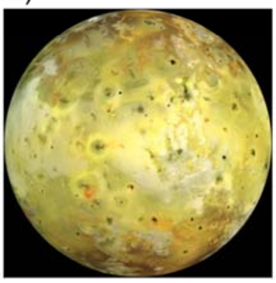


Shoemaker-Levy, Hubble 18 May 1994



Saturn, "Cassini" 27 Mar 2004

Jovian Moon Io, "Galileo" 3 Jul 1999



330 km tall eruption on Io, "New Horizons" 28 Feb 2007

All Images: NASA

Fig. 7: Some further effects of the gravity gradient in our solar system.