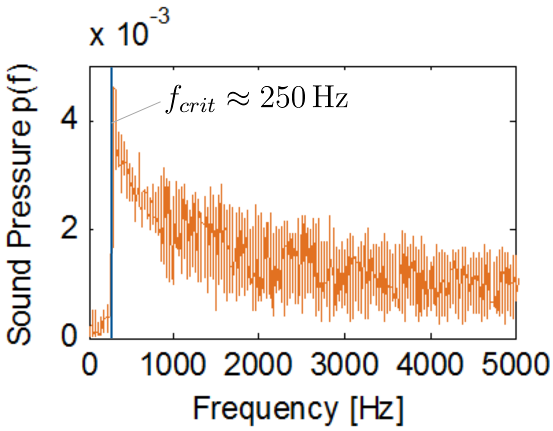
MOOC@TU9 - Models in Civil Engineering: From Cardiology to Fishery
Task of the Week

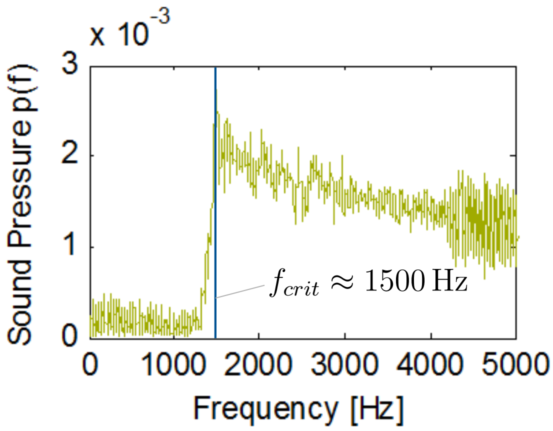
Possible Solution

a) The lowest frequency at which sound is radiated can be read from the sound pressure diagramm in the frequency domain. The frequency drop is clearly visible. The amplitudes below f_{crit} are due to noise in the measurement.

Layer 1:



Layer 2:



After the determination of f_{crit} we have two possibilities to proceed: Either the wavelengths λ or the wave velocities c of the plate and the air at f_{crit} have to be equal.

1. Comparison of wavelengts λ:

$$\lambda_B(f_{crit}) = \lambda_A(f_{crit})$$

$$\Leftrightarrow \quad \sqrt{\frac{2\pi}{f_{crit}}} \cdot \sqrt[4]{\frac{B}{\mu}} = \frac{c_A}{f_{crit}}$$

$$\Rightarrow \quad \sqrt{\frac{2\pi}{f_{crit}}} \cdot \sqrt[4]{\frac{\frac{E \cdot t^3}{12(1-\nu^2)}}{\rho \cdot t}} = \frac{340\,\frac{\text{m}}{\text{s}}}{f_{crit}}$$

$$\Rightarrow \quad t = \sqrt{\left(\frac{340\,\frac{\text{m}}{\text{s}}}{\sqrt{2\pi \cdot f_{crit}}}\right)^4 \cdot \frac{\rho \cdot 12\,(1-\nu^2)}{E}}$$

2. Comparison of wave velocities c:

$$c_B\left(f_{crit}\right) = c_A\left(f_{crit}\right)$$

$$\Leftrightarrow \qquad \sqrt{2\pi \cdot f_{crit}} \cdot \sqrt[4]{\frac{B}{\mu}} = c_A$$

$$\Rightarrow \quad \sqrt{2\pi \cdot f_{crit}} \cdot \sqrt[4]{\frac{\frac{E \cdot t^3}{12(1-\nu^2)}}{\rho \cdot t}} = 340\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$\Rightarrow \qquad t = \sqrt{\left(\frac{340\,\frac{\mathrm{m}}{\mathrm{s}}}{\sqrt{2\pi \cdot f_{crit}}}\right)^4 \cdot \frac{\rho \cdot 12\left(1-\nu^2\right)}{E}}$$

Ase you can see we get the same result as by comparison of the wavelenghts.

Evaluation:

Layer 1:

$$f_{crit,1} \approx 250\,\mathrm{Hz}$$

$$\Rightarrow \; t_1 \approx \sqrt{\left(\frac{340\,\frac{\mathrm{m}}{\mathrm{s}}}{\sqrt{2\pi \cdot 250\,\frac{1}{\mathrm{s}}}}\right)^4 \cdot \frac{916.7\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 12\left(1-0.33^2\right)}{9.1 \cdot 10^9\,\frac{\mathrm{N}}{\mathrm{m}^2}}} = 0.076\,\mathrm{m}$$

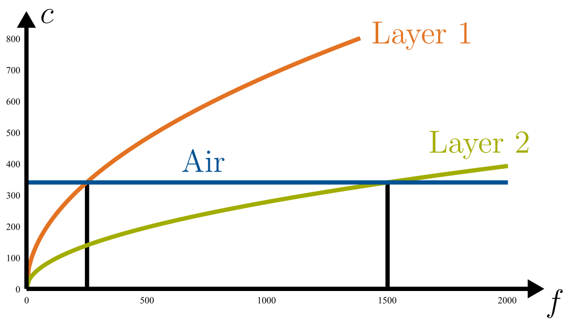
Layer 2:

$$f_{crit,2} \approx 1500\,\mathrm{Hz}$$

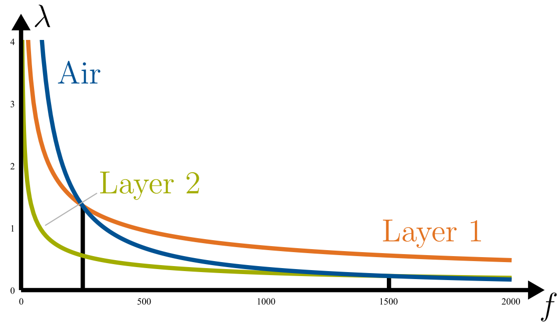
$$\Rightarrow \; t_2 \approx \sqrt{\left(\frac{340\,\frac{\mathrm{m}}{\mathrm{s}}}{\sqrt{2\pi \cdot 1500\,\frac{1}{\mathrm{s}}}}\right)^4 \cdot \frac{916.7\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 12\left(1-0.33^2\right)}{9.1 \cdot 10^9\,\frac{\mathrm{N}}{\mathrm{m}^2}}} = 0.013\,\mathrm{m}$$

Visualization:

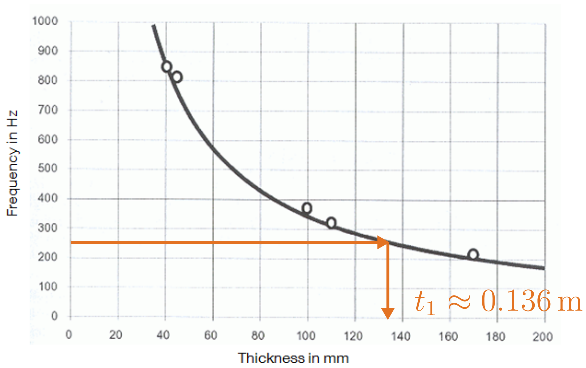
Wave velocity:



Wave length:



b) Comparison with the results published by Lundmark (2001)[1]:



- Comparison possible only for frequencies lower than 1000 Hz
- Thickness of Layer 1 according to Lundmark: $t_1 \approx 0.136\,\mathrm{m}$
- Thickness of Layer 1 according to our estimation: $t_1 \approx 0.076\,\mathrm{m}$

Our plate is less stiff than in reality, because we neglected the support of the water. However, we will always underestimate the thickness of the ice and therefore our estimation is on the „safe side“ concerning the resistance against the jump.

[1] G. Lundmark, „Skating on thin ice - And the acoustics of infinite plates“, Proceedings of the International Congress and Exhibition on Noise Control Engineering, The Hague, 2001