Models in Civil Engineering: From Cardiology to Fishery – Task of the Week (Solution)

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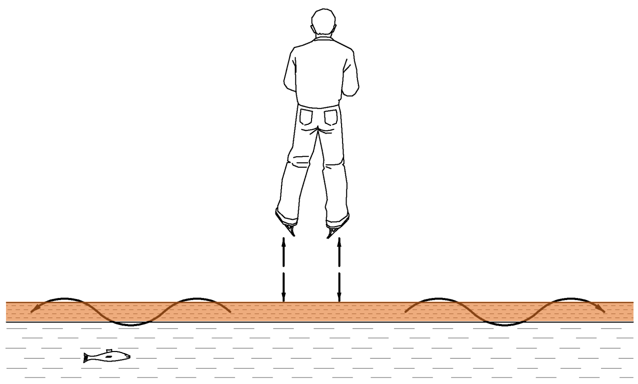
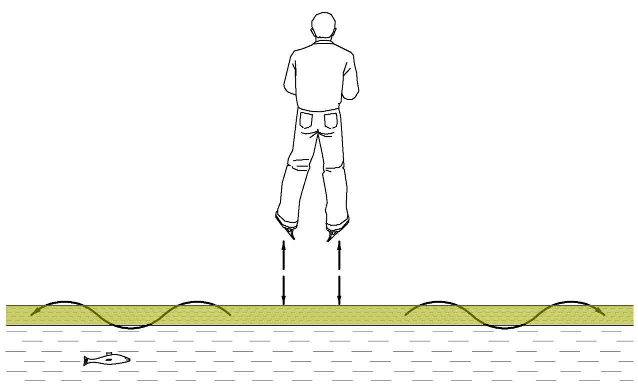
Task of the Week

Given: Results of acoustic measurements for two different ice layers

Layer 1



Layer 2



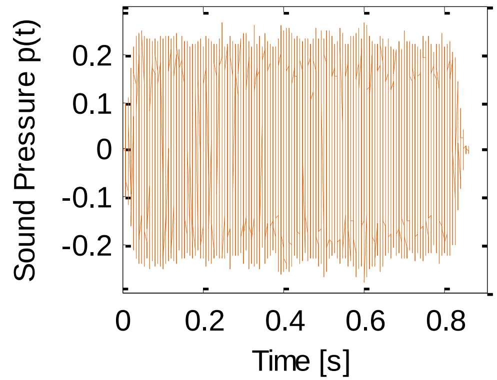
Task of the Week

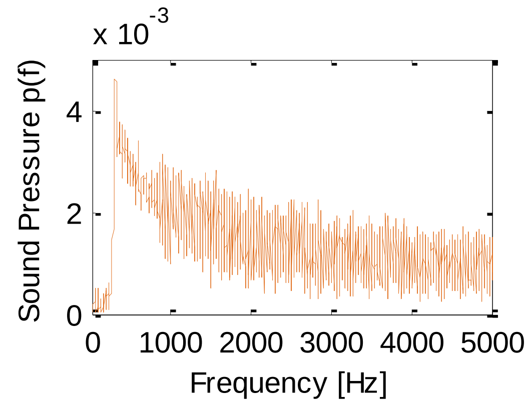
Given: Results of acoustic measurements for two different ice layers

Time Domain

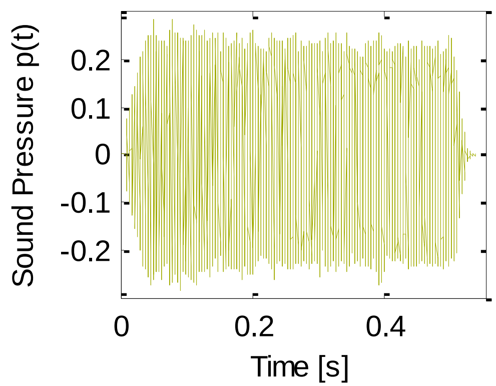
Frequency Domain [0Hz; 5kHz]

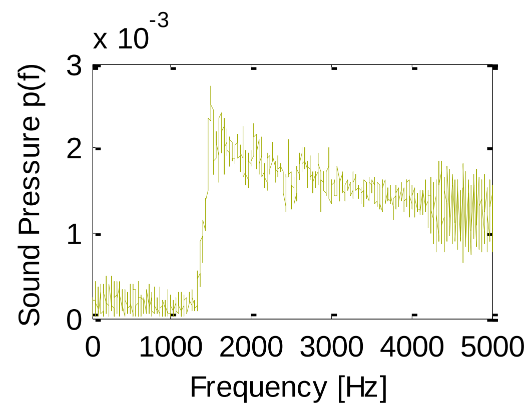
Layer 1





Layer 2





Task of the Week

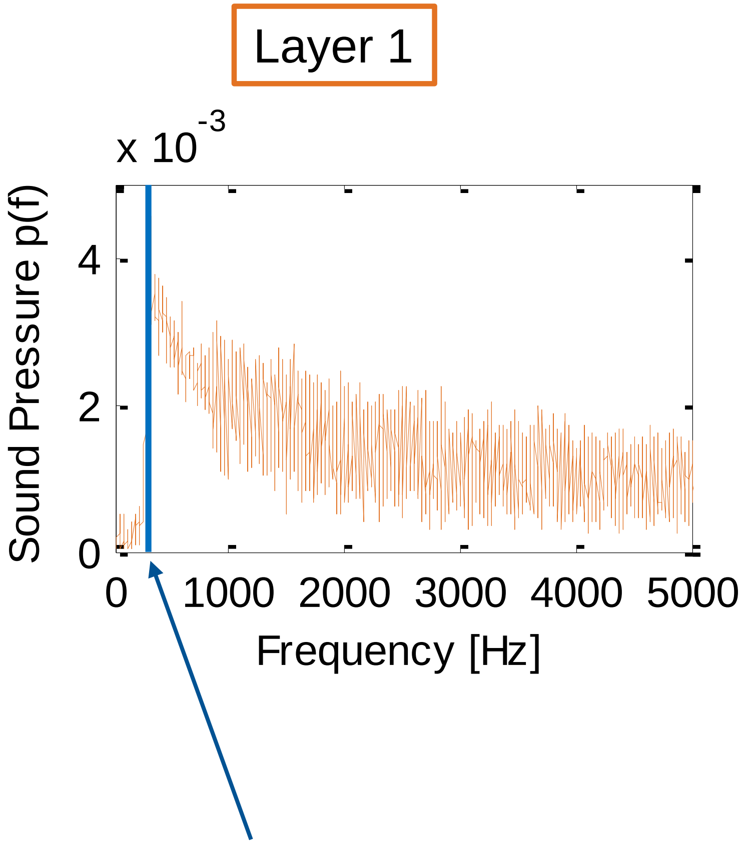
Given: Results of acoustic measurements for two different ice layers

- Compute the thickness of the ice layers (Would you jump on the ice?)
  Use the following material parameters for ice
  - Young's Modulus $E = 9.1 \cdot 10^9 \, \frac{N}{m^2}$
  - Density $\rho = 916.7 \, \frac{kg}{m^3}$
  - Poisson's Ratio $\nu = 0.33$

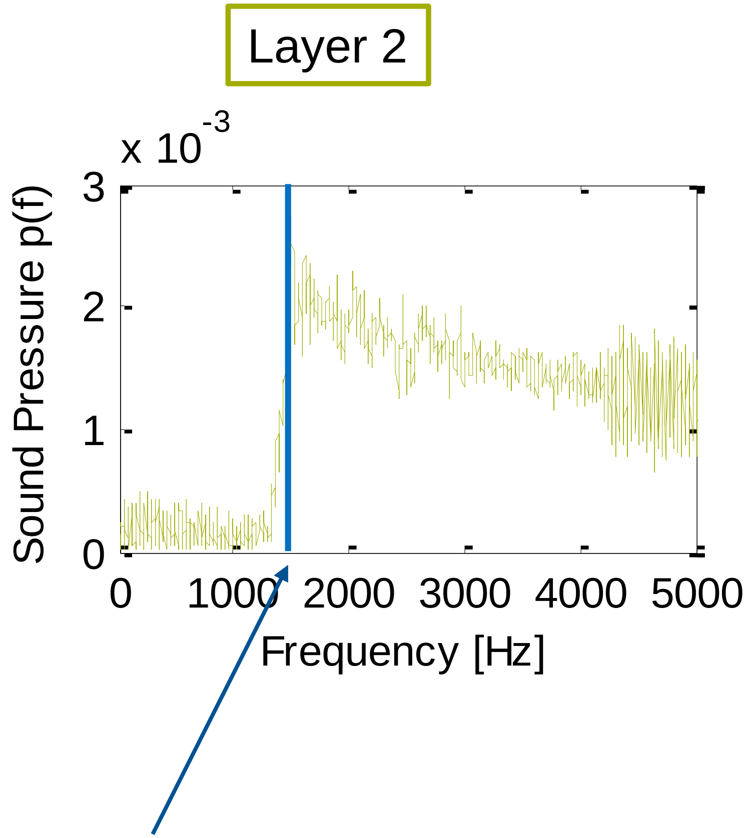
  Use the following constant (frequency independent) wave velocity for air
  - Wave velocity $c_{air} = 340 \, \frac{m}{s}$
- Is the applied model conservative (on the save side)? Please compare your results with the results published by Lundmark (2001) and comment on the differences. Under what circumstances would you trust the results in order to jump on the ice?
- Estimate the stiffness of the water using the model of an elastically supported plate. Comment on the model of an elastic support of the plate.

Task of the Week

Possible solution



$f_{crit,1} \approx 250\ Hz$



$f_{crit,2} \approx 1500\ Hz$

Task of the Week

Possible solution

Two possibilities: either the wave lengths λ or the wave velocities c of the plate and the air have to be equal

1. Comparison of wave lengths λ:

$$\lambda_B(f_{crit}) = \lambda_{air}(f_{crit})$$

$$\sqrt{\frac{2\pi}{f_{crit}}} \cdot \sqrt[4]{\frac{B}{\mu}} = \frac{c_{air}}{f_{crit}}$$

$$\sqrt{\frac{2\pi}{f_{crit}}} \cdot \sqrt[4]{\frac{\frac{E \cdot t^3}{12(1-\nu^2)}}{\rho \cdot t}} = \frac{340\,\frac{m}{s}}{f_{crit}}$$

$$\Rightarrow t = \sqrt{\left(\frac{340\,\frac{m}{s}}{\sqrt{2\pi \cdot f_{crit}}}\right)^4 \cdot \frac{\rho \cdot 12(1-\nu^2)}{E}}$$

Task of the Week

Possible solution

2. Comparison of wave velocity c:

$$c_B(f_{crit}) = c_{air}$$

$$\sqrt{2\pi \cdot f_{crit}} \cdot \sqrt[4]{\frac{B}{\mu}} = c_{air}$$

$$\sqrt{2\pi \cdot f_{crit}} \cdot \sqrt[4]{\frac{\frac{E \cdot t^3}{12(1-\nu^2)}}{\rho \cdot t}} = 340\,\frac{m}{s}$$

$$\Rightarrow t = \sqrt{\left(\frac{340\,\frac{m}{s}}{\sqrt{2\pi \cdot f_{crit}}}\right)^4 \cdot \frac{\rho \cdot 12(1-\nu^2)}{E}}$$

→ Same result as by comparison of wave lengths

Task of the Week

Possible solution

Layer 1

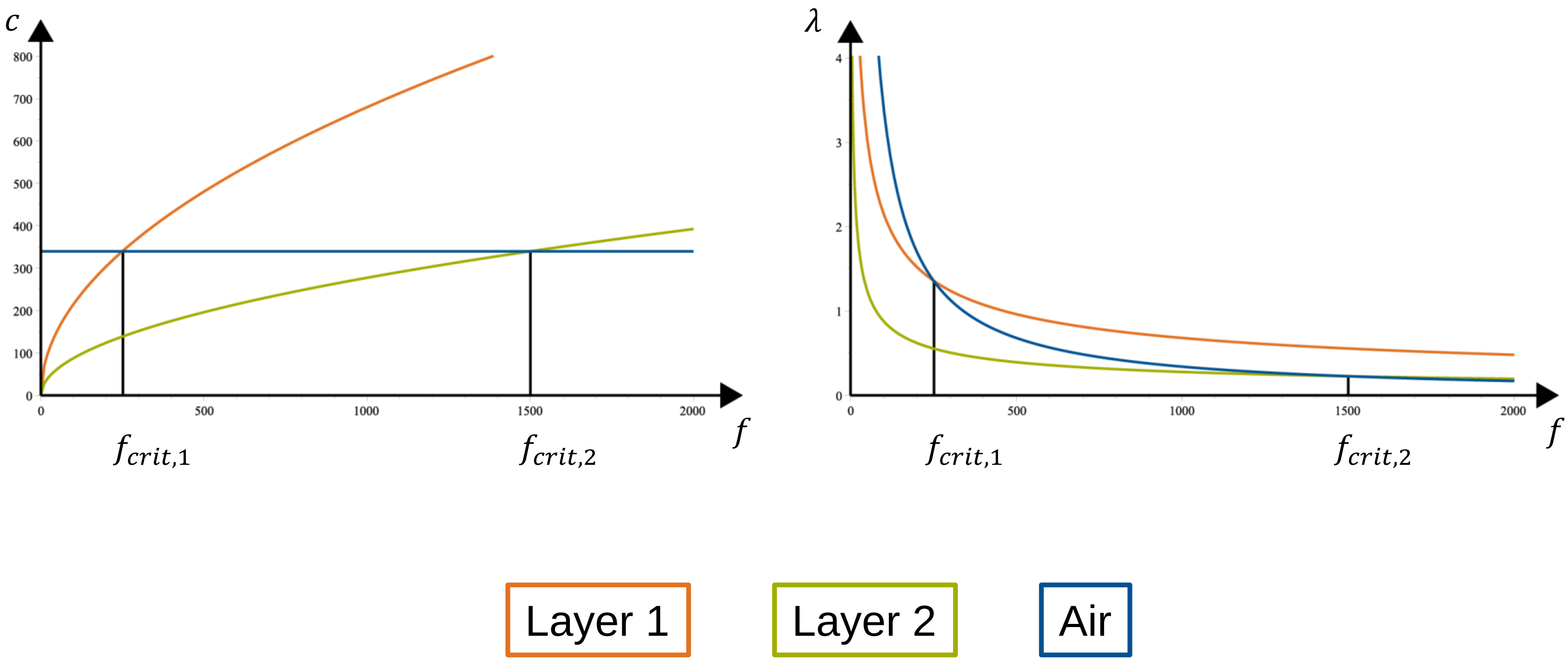
$$f_{crit,1} \approx 250\ Hz \Rightarrow t_1 \approx \sqrt{\left(\frac{340\,\frac{m}{s}}{\sqrt{2\pi \cdot 250\,\frac{1}{s}}}\right)^4 \cdot \frac{916.7\,\frac{kg}{m^3} \cdot 12(1-0.33^2)}{9.1 \cdot 10^9\,\frac{N}{m^2}}} = 0.076\ m$$

Layer 2

$$f_{crit,2} \approx 1500\ Hz \Rightarrow t_1 \approx \sqrt{\left(\frac{340\,\frac{m}{s}}{\sqrt{2\pi \cdot 1500\,\frac{1}{s}}}\right)^4 \cdot \frac{916.7\,\frac{kg}{m^3} \cdot 12(1-0.33^2)}{9.1 \cdot 10^9\,\frac{N}{m^2}}} = 0.013\ m$$

Task of the Week

Possible solution

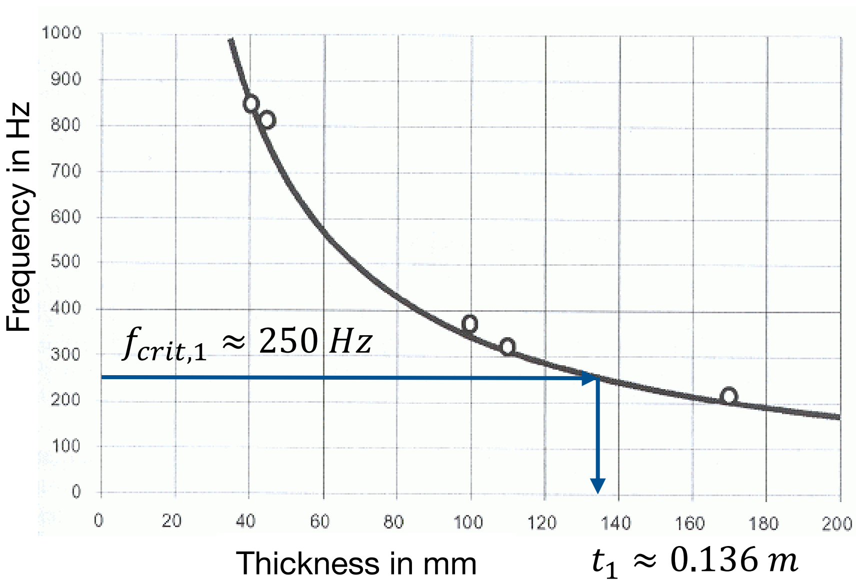


Task of the Week

Possible solution

Comparison with the results published by Lundmark, 2001:

- Comparison possible only for frequencies lower than 1000 Hz
- Thickness of Layer 1 according to Lundmark: $t_1 \approx 0.136\,m$
- Thickness of Layer 1 according to our estimation: $t_1 \approx 0.076\,m$



- Due to the neglecting of the support of the water our plate is less stiff than in reality
- Estimation on the „safe“ side concerning the resistance against a jump