Orbital Mechanics – Basics

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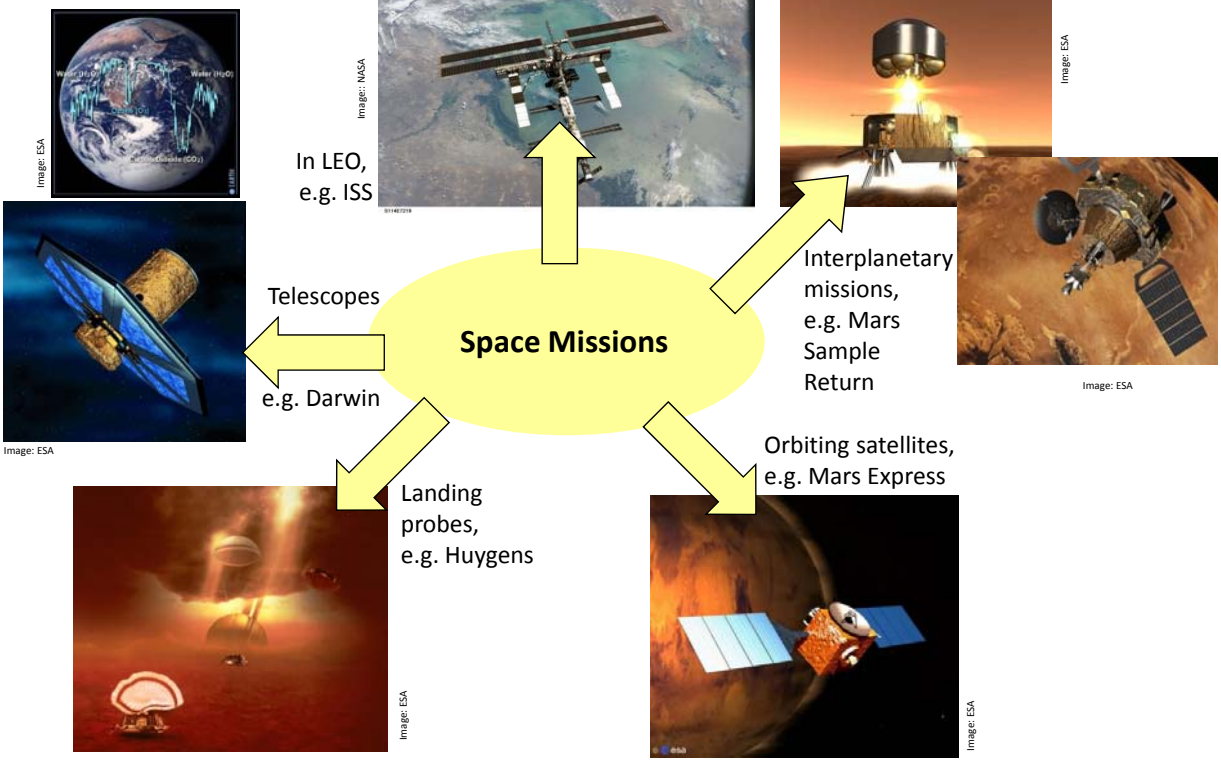
In the frame of the MOOC@TU9 Lecture Course Network

Stuttgart, December 15th, 2014

Introduction



Lesson Objectives:
Answers to Typical Fundamental Questions…

- Why does a satellite stay in orbit???
- Is an astronaut weightless???
- Which orbit types can be realized???
- Which velocities do we need???
- Do we "feel" on Earth the gravitational pull of the Sun???
  (mostly as "task of the week"…)

Newton's Laws

1687: **Isaac Newton** formulates the laws of motion:

1. *Unless acted upon by an unbalanced force, an object will maintain a constant velocity (magnitude and direction).*
2. *An applied force is equal to the rate of change of momentum.*
3. *All forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.*

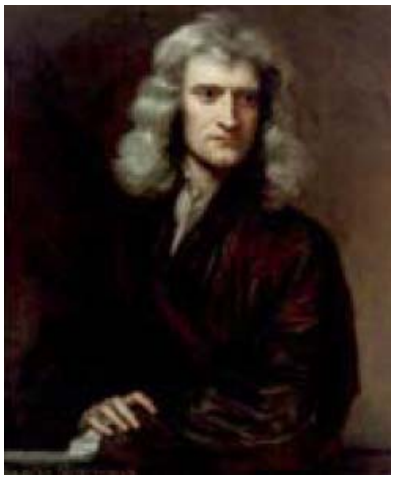


Photo: http://www.newton.org.uk/gallery/kn89-1.jpg

Newton's law of universal gravitation states:

Every point mass attracts every other point mass by a force directed along the line connecting the two. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them:

$$\vec{F}_{on\ m_2} = -\gamma \frac{m_1 m_2}{r_{1\to 2}^2} \frac{\vec{r}_{1\to 2}}{r_{1\to 2}} \qquad F = -\gamma \frac{m_1 m_2}{r^2}$$

$$\gamma = 6.674 \cdot 10^{-11} \frac{m^3}{kgs^2} (\pm 0.01\%)$$

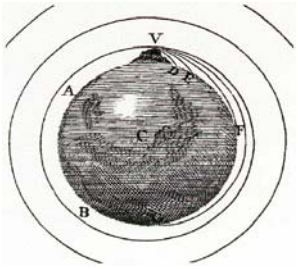


Image: http://galileoandeinstein.physics.virginia.edu/lectures/newtmtn.gif

Explanations why a Satellite Stays in Orbit

- Gravitational force is equal to centrifugal force…

  Easy to understand, but physically wrong…Why?

  → Newton's 1st law,

  i.e. no constant velocity in magnitude and direction!

- Gravity accelerates the satellite towards Earth, however because of its velocity it will never touch the Earth's surface, i.e.

  satellite is in a continuous free fall „around" the Earth…

  (the correct answer, at least up to the times of Albert Einstein…)

- Remark:

  Relativistic: Gravity mass causes a curvature in space-time, in which the satellite is caused to move…

  (far beyond scope of this lecture…)

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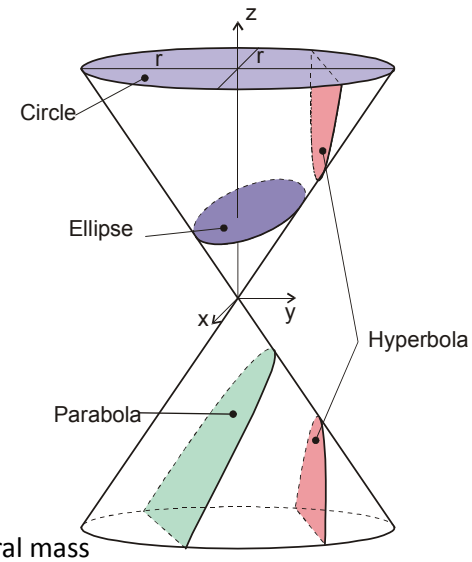
Gravitational Fields: General solution

Basic equation in orbital mechanics:
Vis-Viva-Equation or energy conservation equation:

$$\underbrace{\frac{1}{2}v^2}_{\text{kinetic energy}} \underbrace{-\frac{\gamma M}{r}}_{\text{potential energy}} = const. = -\frac{\gamma M}{2a} = \frac{1}{2}v_\infty^2$$



Solutions: Conic sections, i.e.:
- Ellipse (special case: circle),
- Parabola,
- Hyperbola.

Physical meaning:
- Ellipse, circle: Closed orbit around the central mass (a = semi-major axis, $a = r$ for circular orbits)
- Parabola: Velocity high enough to escape from central mass, having a zero velocity relative to this mass in "infinity" ($v_\infty = 0$)
- Hyperbola: Velocity high enough to escape from central mass, having a finite velocity relative to this mass in "infinity" ($v_\infty > 0$)

Circular Orbits

From the Vis-Viva equation with $a = r$:

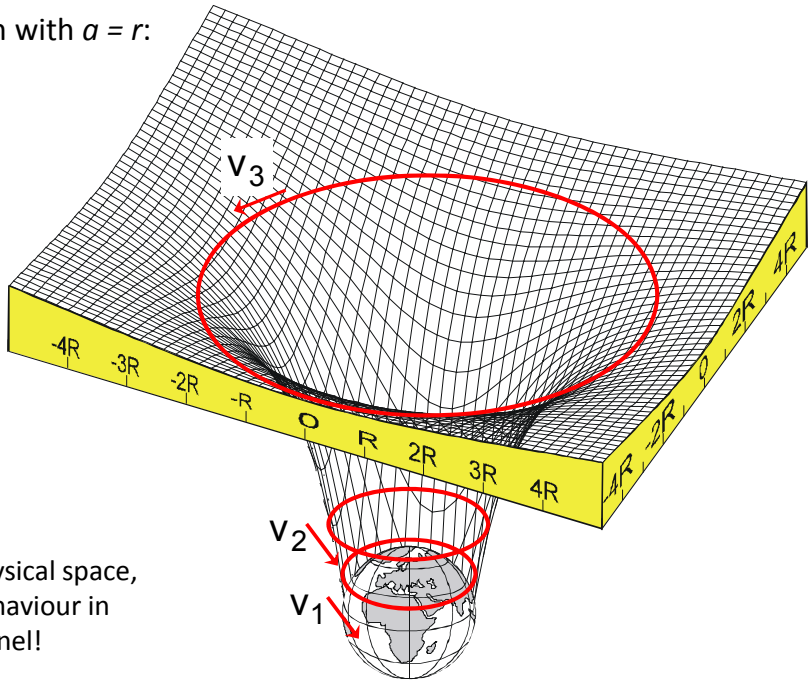
$$v_{circle} = \sqrt{\frac{\gamma M}{r}}$$

Thus

$$v_1 > v_2 > v_3$$

because

$$r_1 < r_2 < r_3$$

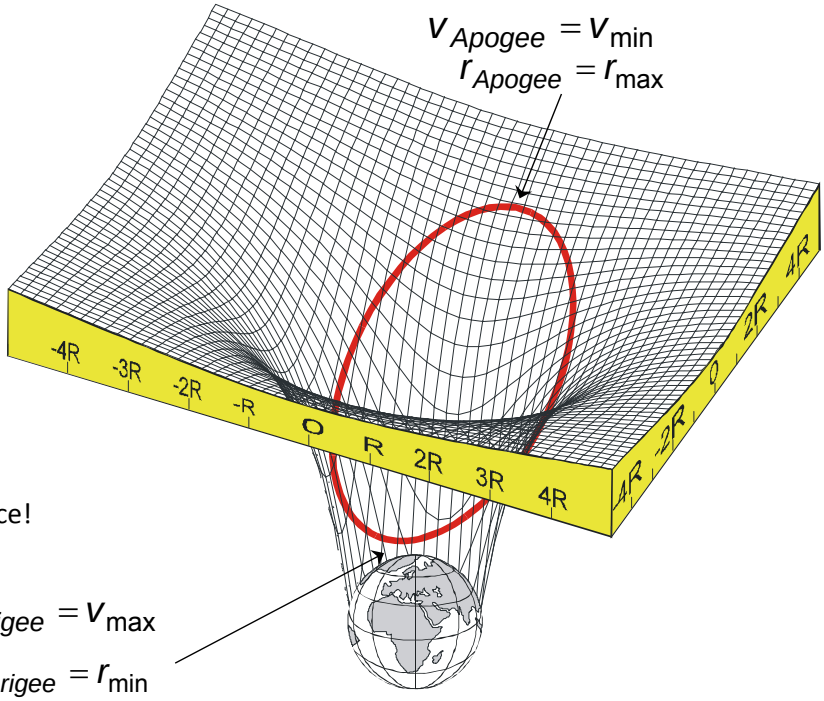


Caution:
These are **not** orbits in physical space, illustrated is just the behaviour in a gravitational funnel!

Elliptical Orbits

Elliptical orbits:

$$v = \sqrt{\frac{2\gamma M}{r} - \frac{\gamma M}{a}}$$



Caution:
This is **not** an orbit in physical space!

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Some Velocities

From the Vis-Viva equation with $a = r = R_{Earth}$:

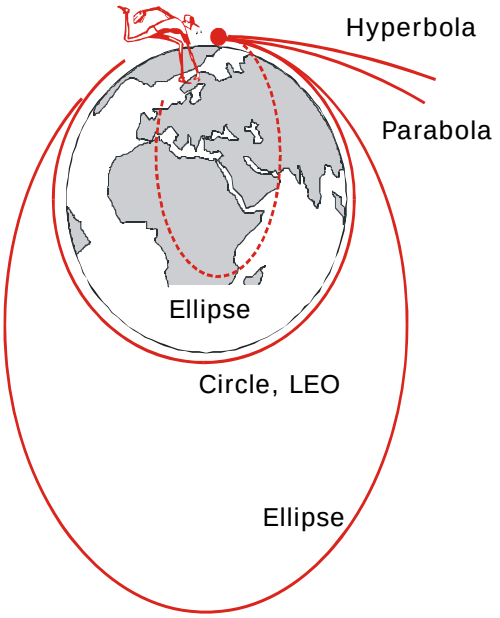
$$v_{LowEarthOrbit} \approx \sqrt{\frac{\gamma M_{Earth}}{R_{Earth}}} \approx 7.91 \frac{km}{s}$$

$$M_{Earth} \approx 6 \cdot 10^{24} kg, \quad R_{Earth} \approx 6378 km$$

For a parabola:

$$v = \sqrt{\frac{2\gamma M}{r} - \frac{\gamma M}{a}} \qquad a \to \infty$$

$$v_{Parabola} \approx \sqrt{\frac{2\gamma M}{r}} \underset{\text{if Earth surface}}{=} \sqrt{\frac{2\gamma M_{Earth}}{R_{Earth}}} \approx 11.2 \frac{km}{s}$$



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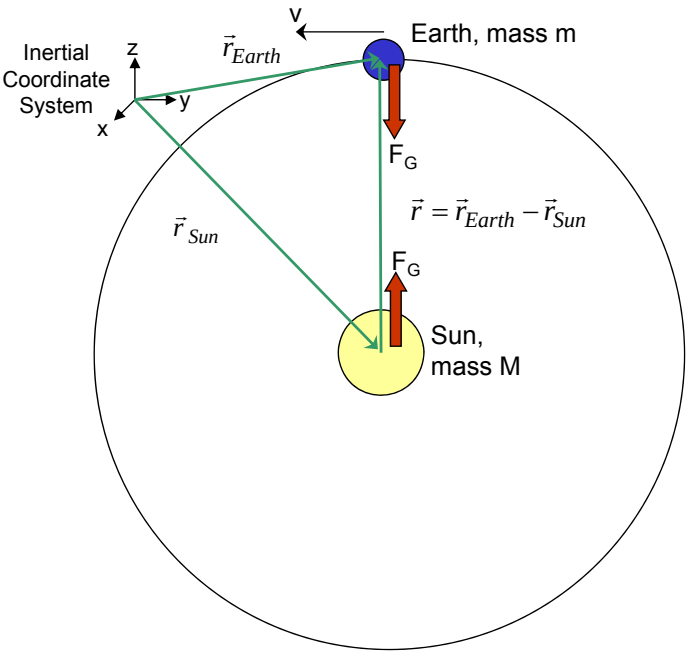
The 2-Body-Problem for the Sun-Earth-System



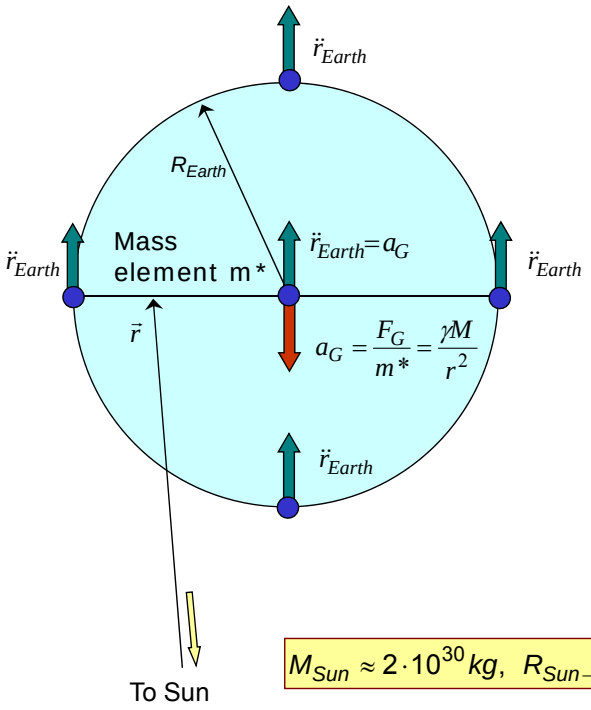
- Gravitational pull on Earth is compensated by a continuous „free-fall“ towards Sun.
- Acceleration occurs (almost) perpendicularly to Earth's own velocity, changing continuously the direction of the velocity vector.
- Acceleration is thereby so that the amount of Earth's velocity remains (almost) unchanged.

⇒ ***The entire Earth is in a continuous free-fall around the Sun!***

Assumption here: Point mass!!!

⇒ ***What happens if we consider a three-dimensional Earth??***
⇒ ***"task of the week"...***

Task of the Week - Introduction
Do we "feel" on Earth the gravitational pull of the Sun???



$M_{Sun} \approx 2 \cdot 10^{30}\,kg, \quad R_{Sun \to Earth} = r \approx 150 \cdot 10^{9}\,m$

$M_{Earth} \approx 6 \cdot 10^{24}\,kg, \quad R_{Earth} \approx 6378\,km$