Topic: Massive MIMO

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MOOC @ TU9

Week 3

Task of the Week – Solutions

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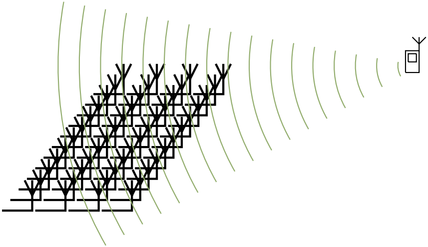
Channel State Information (CSI) Acquisition

Uplink Training

$$\hat{\boldsymbol{h}} = \boldsymbol{h} \qquad + \boldsymbol{n}$$



Channel State Information (CSI) Acquisition

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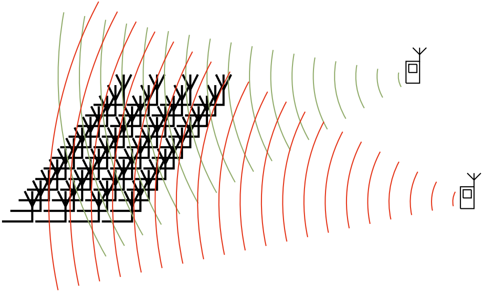

Pilot-Contamination

Questions

- What is the impact of PILOT CONTAMINATION (PC) on the downlink communication mode?
- What kind of COUNTERMEASURES (against PC) could be thought of?

- **Hint**: Compute the SINR at the i-th receiver based on the erroneous channel vector estimates

Solutions

Due to PC we obtain the corrupted channel estimate $\hat{\boldsymbol{h}}_i = \boldsymbol{h}_i + \boldsymbol{h}_{I,i}$[1] and consequently the beamformer $\boldsymbol{w}_i = \gamma_i^{-1}(\boldsymbol{h}_i + \boldsymbol{h}_{I,i})^*$.[2] **Note**: due to PC $\boldsymbol{h}_{I,i}$ contains a mixture of all channel vectors $\boldsymbol{h}_{j\neq i}$.

The erroneous beamformer leads to a non-zero interference term in the received signal y_i at the i-th receiver even in case that $\boldsymbol{h}_i^{\mathrm{T}}\boldsymbol{h}_j^*/M \to 0$ for $i \neq j$:

$$\begin{aligned} y_i &= \boldsymbol{h}_i^{\mathrm{T}} \left(\sum_{j=1}^{K} \boldsymbol{w}_j s_j \right) + n_i \\ &= \gamma_i^{-1} \boldsymbol{h}_i^{\mathrm{T}} (\boldsymbol{h}_i + \boldsymbol{h}_{I,i})^* s_i + \sum_{\substack{j=1 \\ j\neq i}}^{K} \gamma_j^{-1} \boldsymbol{h}_i^{\mathrm{T}} (\boldsymbol{h}_j + \boldsymbol{h}_{I,j})^* s_j + n_i. \end{aligned}$$

[1] We neglect the channel estimation noise, i.e. we assume perfect channel estimation.

[2] γ_i stands for the $\|\boldsymbol{h}_i + \boldsymbol{h}_{I,i}\|$.

For $M \to \infty$, we learned that $\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{h}_j^* / M \to 0$. Therefore

$$y_i \quad \to \quad \gamma_i^{-1} \|\boldsymbol{h}_i\|^2 s_i + \underbrace{\gamma_i^{-1} \boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{h}_{I,i}^*}_{\text{A}} s_i + \sum_{\substack{j=1 \\ j \neq i}}^{K} \gamma_j^{-1} \underbrace{\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{h}_{I,j}^*}_{\text{B}} s_j + n_i.$$

Term A again tends to zero ($\boldsymbol{h}_{I,i}$ only contains $\boldsymbol{h}_{j \neq i}$), but the terms in B leads to an interference of the data stream to the i-th receiver by signals dedicated to other receivers ($\boldsymbol{h}_{I,j}$ contains $\boldsymbol{h}_i$). The respective SINR can be obtained as

$$\text{SINR} \quad \to \quad \frac{\gamma_i^{-2} \|\boldsymbol{h}_i\|^4 \sigma_{s_i}^2}{\sum_{\substack{j=1 \\ j \neq i}}^{K} \gamma_j^{-2} \left| \boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{h}_{I,j}^* \right|^2 \sigma_{s_j}^2 + \sigma_{n_i}^2}.$$

As a consequence of PC, the SINR cannot grow linearly with M, even saturates and the benefits of Massive MIMO are destroyed.

Solutions (cont'd)

In order to neglect the estimation error $\boldsymbol{h}_{I,i}$, the training sequences, which are sent from the users to the basestation, are designed to be orthogonal to each other.

To this end, each user sends a training sequence (pilot data) $\boldsymbol{s}_i^{\mathrm{T}} = [s_{i,1}, s_{i,2}, \ldots, s_{i,N}]$ of length $N \geq K$, with $\boldsymbol{S} = [\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_K]^{\mathrm{T}}$ and $\boldsymbol{S}\boldsymbol{S}^{*,\mathrm{T}} = \boldsymbol{I}$.[3]

The basestation receives the spatio-temporal signal

$$\boldsymbol{h}_i \boldsymbol{s}_i^{\mathrm{T}} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \boldsymbol{h}_j \boldsymbol{s}_j^{\mathrm{T}},$$

where channel estimation noise is neglected again.

[3] $\boldsymbol{I}$ is the identity matrix.

Solutions (cont'd)

Due to their mutual orthogonality, the training sequences can be separated from each other at the transmitter and $\boldsymbol{h}_{I,i}$ can be suppressed:

$$\left(\boldsymbol{h}_i \boldsymbol{s}_i^{\mathrm{T}} + \sum_{\substack{j=1 \\ j \neq i}}^{K} \boldsymbol{h}_j \boldsymbol{s}_j^{\mathrm{T}} \right) \boldsymbol{s}_i^* = \boldsymbol{h}_i .$$

A special case would be a protocoll where alternately a single user sends pilot data and the others are idle.

Solutions (cont'd)

Although the proposed solution seems to be an appropriate countermeasure agains PC, its meaning is limited due to a couple of reasons:

i. requires synchronization among users and/or orthogonality between shifted training sequences

ii. runs out of sequences for many users in the communication cell

iii. fails in suppressing PC caused by active users in neighboring communication cells, etc.

Therefore more advanced schemes have been proposed.

Solutions (cont'd)

More advanced schemes against PC can be categorized into a few directions: among them are 1) advanced channel estimation schemes and 2) cooperative precoding among multiple transmitters.

The first direction is based on channel estimation taking into account prior information at the transmitter (e.g., stochastic properties) of all communication channels in order to reduce the effects of the PC phenomenon.

The precoding approach considers an advanced system design that requires a cooperation among multiple transmitters and the knowledge of stochastic properties of all communication channels (variance of the channel coefficients) that can be overheard at the basestation.

Solutions (cont'd)

Additionally, a more advanced understanding of the massive MIMO effect is required, namely that the inner product of to channel vectors, i.e.

$$\boldsymbol{h}_i^{\mathrm{T}} \boldsymbol{h}_j^* / M \rightarrow \begin{cases} 0 & ; \quad i \neq j \\ \sigma_{h_i}^2 & ; \quad i = j, \end{cases}$$

tends to zero for $M \rightarrow \infty$ for $i \neq j$, but otherwise converges to the variance $\sigma_{h_i}^2$ of the channel coefficients of $\boldsymbol{h}_i$.

Therefore the transmitters knowing in advance the effective desired channels and cross channels (given by the stochastic properties) can cooperate to eliminate the interference effect while transmitting data to the receivers.

Outlook

If you got interested in this topic you can learn more about WIRELESS COMMUNICATIONS, 5G, MASSIVE MIMO, PILOT CONTAMINATION, COOPERATIVE COMMUNICATIONS, PRECODING, etc. by searching for this terms in the world wide web and at `ieeexplore.ieee.org`

You might also by interested in joining TUM in a master's program such as `master.ei.tum.de`